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# A New Mathematical Method Of Image Denoising Using Partial Differential Equations

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***Abstract:*** *Many methods like total variation, nonlocal means and wavelet method are used for remote sensing image denoising. All these denoising methods depends on the features that all come from the noisy image itself. In fact, the image features acquired by other sensors from the same scene can also be used as priors in denoising. So, a new method for denoising remote sensing images based on partial differential equations (PDEs) is proposed. It uses the features such as direction of smoothening and strength of smoothening of the auxiliary image too. The auxiliary image is both similar to and different from the noisy image. The image intensity distributions are different, but the edge directions and texture information are similar. The correlation between the different bands of multicomponent images is used. The auxiliary image as the priories introduced into the TV or partial differential equation (PDE) denoising method. Moreover, the auxiliary image is applied in the form of a noise-free single-component image.*

***Keywords:*** *Partial differential equations (PDE), auxiliary image, wavelet transform, Multicomponent image.*

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## I. Introduction

In this paper, a new method for denoising remote-sensing images based on partial differential equations (PDEs) is proposed. The method employs the similarity between the different band images in a multicomponent image. Initially, one of the noise-free images in multicomponent remote-sensing images as a prior is introduced into the PDE denoising method. To make use of the priors of the noise-free image in denoising, here construct a new smoothing term for the PDE to compute the total variation. The new smoothing term refers to a specific smoothing direction and a specific smoothing intensity of the reference image when denoising the noisy image. The proposed smoothing term is added as a new constraint into the PDE denoising method. Based on the proposed method, the similarity of the directions of the edges between the noisy image and the reference image enables the new algorithm to smooth. out more noise and conserve more detail in the denoising process. We also present the discrete form of the proposed denoising model. Multispectral remote-sensing images and hyperspectral remote-sensing images are experimented in this letter. A better performance is achieved by the proposed method when compared with other methods. In recent years, several classes of denoising algorithms such as total variation (TV)[13], wavelets[14], and nonlocal means[1] have all achieved much success. These algorithms are based on different theories, and all show good performance in denoising. When denoising an image, the TV method makes use of the geometric features of the image, the wavelet method makes use of the statistical features of the coefficients, and the nonlocal means method makes use of the redundancy in the image texture features. However, the features that have been used by these methods all come from the noisy image itself. In fact, the image features acquired by other sensors from the same scene can also be used as priors in denoising. In many situations involving multicomponent remote- sensing images, a single-component image with a higher SNR or higher spatial resolution is often available. In the past, such an auxiliary image was applied for fusion with a multispectral image to improve its spatial resolution. In fact, the auxiliary noise-free image is a more suitable aid to denoising. An auxiliary image as a prior is used to assist in denoising or deblurring the image, but this is not suitable for a remote-sensing image, and the auxiliary noisy image must come from the same sensor. There has been some research on denoising based on such an auxiliary image in the remote- sensing field. In hyperspectral images, the infrared part of the spectrum contains noise near the water-vapor absorption band. To denoise these bands, image bands from other parts of the spectrum can be applied as noise free images. Recently, a multispectral and hyperspectral image denoising algorithm has been proposed and has achieved good results, where within the Bayesian framework, the extra initial information is included in the form of a noise-free single-band image. The goal of our approach is denoising. The basic idea behind this project is to denoise the multicomponent remote sensing images using partial differential equation using auxiliary images as priors. The project has a wide area of applications in hyperspectral and multispectral remote sensing image denoising.

## II. Partial differential equations

The space of all PDEs is infinite dimensional. To find the right form, we start with the properties that our PDE system should have, to narrow down the search space. We notice that most image processing problems are shift and rotationally invariant, i.e., when the input image is shifted or rotated, the output image is also shifted or rotated by the same amount. So, we require that our PDE system is shift and rotationally invariant. In the work of Alvarezet al, different forms of PDEs are determined by assuming various transformational invariance.

Then according to the differential invariant theory, Lo and L_ must be functions of the fundamental differential invariants under the groups of shift and rotation. The fundamental differential invariants are invariant under shift and rotation and other invariants can be written as their functions. The set of fundamental differential invariants is not unique, but different sets can express each other. We should choose invariants in the simplest form in order to ease

mathematical deduction and analysis and numerical computation. Fortunately, for shift and rotational invariance, the fundamental differential invariants can be chosen as polynomials of the partial derivatives

### A. A PDE-Based Approach to Three-Dimensional Seismic Data Fusion

Sorin Pop, Olivier Lavialle, and Marc Donias presented a new method for the denoising and fusion, which is dedicated to multi azimuth seismic data in 2008.It proposed to combine low-level fusion and diffusion processes through the use of a unique model based on partial differential equations. The denoising process is driven by the seismic fault preserving diffusion equation. Meanwhile, relevant information (as seismic faults) is injected in the fused 3-D images by an inverse diffusion process. One of the advantages of such an original approach is to improve the quality of the results in case of noisy inputs, which are frequently occurring in seismic unprocessed data. The acquisition and processing of reflection seismic data result in seismic blocks (i.e.,3-D images) of acoustic impedance interfaces. The interpretation of these data represents a delicate task. Geological patterns are often difficult to recognize for the expert. This interpretation of seismic blocks mainly consists in reflector picking (i.e., identifying and recording the position of specific reflection events) and fault plane locating. As manual interpretation is both costly and subjective, some authors have investigated the use of image processing to develop automatic approaches. The resulting automatic tools are useful for structural interpretation of seismic data, but these tools failed in tracking horizons across faults particularly if the level of noise is high. Finally, most approaches are semiautomated because of the difficulty in dealing with subtle features. One way to improve the efficiency of manual, semiautomated, or automatic interpretation is to increase the quality of the 3-D seismic data by enhancing the horizons and preserving the faults. The interest of multi azimuth (MAZ) acquisition of 3-D reflection seismic data has been proved during the last few years. The MAZ consists in combining several conventional narrow-azimuth surveys over the same area at regular increments of azimuth and is particularly attractive in case of small signal-to-noise ratio (SNR) and poor illumination.

The combination of the information provided by the different surveys (i.e., different azimuths) can be achieved by means of a simple stack or can be viewed as a classical fusion problem. The authors highlight the interest of an optimized stack or fusion in comparison with a conventional stack using all azimuths. Different processing techniques have been proposed in the recent past to optimally extract the relevant signal and reject noise from each azimuth. In this paper, starting from the set of k blocks corresponding to the k azimuths, we propose to use an image fusion approach to obtain a final result including the relevant information about seismic events in general and faults in particular. Image fusion is a process consisting in combining different registered images to increase the quality of the resulting images. In case of pixel-level fusion, the value of the pixels in the fused image is determined from a set of pixels in each input image. In order to obtain output images that contain better information, the fusion algorithms must fulfil some requirements: the algorithm must not discard the relevant

information contained in the input images. Additionally, it must not create any artifacts or inconsistencies in the output images. Over the past two decades, many works were dedicated to image-level fusion methods. The proposed approaches range from simple spatial image fusion by different weighted combinations to complex multiresolution pyramid and wavelet methods It proposed an original approach based on the use of partial differential equations (PDEs). This approach is totally different from the techniques. The basic idea is to combine pixel-level fusion and diffusion processes through one single powerful equation. The insertion of the relevant information contained in inputs is achieved in the fused output by reversing the diffusion process. The diffusion term allows one to reject the noise from each input during the fusion process.

## III. Proposed method

In many situations involving multicomponent remote sensing images, a single-component image with a higher SNR or higher spatial resolution is often available. In the past, such an auxiliary image was applied for fusion with a multispectral image to improve its spatial resolution. In fact, the auxiliary noise-free image is a more suitable aid to denoising. In past, an auxiliary image as a prior is used to assist in denoising or deblurring the image, but this is not suitable for a remote-sensing image, and the auxiliary noisy image must come from the same sensor. There has been some research on denoising based on such an auxiliary image in the remote sensing field. In hyperspectral images, the infrared part of the spectrum contains noise near the water-vapor absorption band. To denoise these bands, image bands from other parts of the spectrum can be applied as noise-free images. Recently, a multispectral and hyperspectral image denoising algorithm has been proposed [8] and has achieved good results, where within the Bayesian framework, the extra initial information is included in the form of a noise-free single-band image. The goal of our approach is denoising. Here the correlation between the different bands of multicomponent images is used. The auxiliary image as the prior is introduced into the TV or partial differential equation (PDE) denoising method. Moreover, the auxiliary image is applied in the form of a noise-free single-component image (no image is completely noise-free and by noise-free we mean with a high SNR). To illustrate the proposed method, we experiment on the multispectral and hyperspectral remote sensing images. A new method for denoising remote-sensing images based on partial differential equations (PDEs) is proposed. The method employs the similarity between the different band images in a multicomponent image. Initially, one of the noise-free images in multicomponent remote-sensing images as a prior is introduced into the PDE denoising method. To make use of the priors of the noise-free image in denoising, we construct a new smoothing term for the PDE to compute the total variation. The new smoothing term refers to a specific smoothing direction and a specific smoothing intensity of the reference image when denoising the noisy image. The proposed smoothing term is added as a new constraint into the PDE denoising method. Based on the proposed method, the similarity of the directions of the edges between the noisy image and the reference image enables the new algorithm to smooth out more noise and conserve more detail in the denoising process. We also present the discrete form of the proposed denoising model. Multispectral remote-sensing images and hyperspectral remote-sensing images are experimented in this letter. A better performance is achieved by the proposed method when compared with other methods.

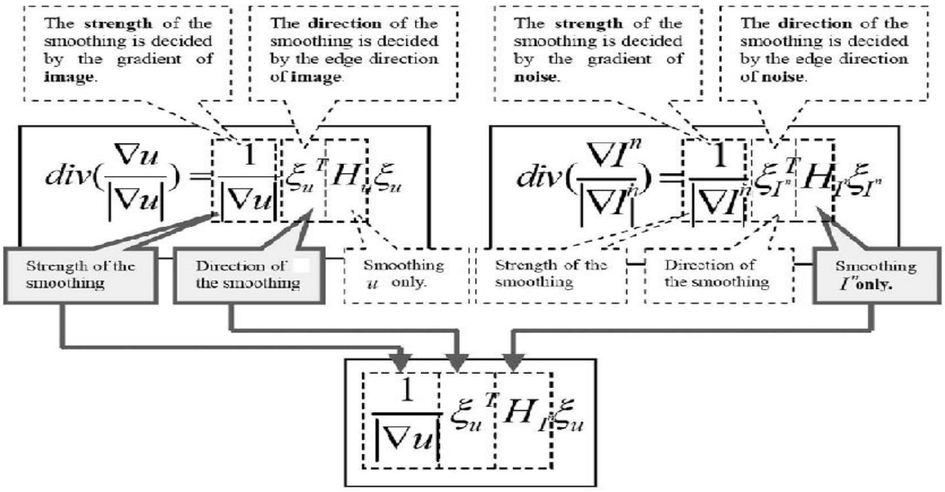


**Figure 1**

### A. Solution to the problem

The goal of our approach is denoising. In this letter, following the correlation between the different bands of multicomponent images are used. The auxiliary image as the prior is introduced into the TV or partial differential equation (PDE) denoising method. Moreover, the auxiliary image is applied in the form of a noise-free single-component image (no image is completely noise-free and by noise-free we mean with a high SNR). To illustrate the proposed method, we experiment on the multispectral and hyperspectral remote sensing images

### B Image denoising based on Partial differential Equations

When an image is corrupted by noise, the following is used:

. I0 = I + n:-----------------------(1)

I0is the observed image, I is the original image, and n is the additive noise in the observed image. Usually, n is assumed to follow a Gaussian distribution with a zero mean and a variance of The TV denoising model[3] is denoted by

$$TV(I) = \int_S |\nabla I| \, dxdy + \lambda/2 \int_s (I - I_0)^2 dxdy$$

reducing an auxiliary image as a prior into pde denoising In many situations in the remote-sensing area, multicomponent images are often acquired. Although an image comprised of several bands is corrupted by noise, a single-component image with a higher SNR is often available. For multispectral and hyperspectral images, there are

often noise-free image bands that can be used as priors in the denoising process. In this letter, the auxiliary image from another sensor is denoted as the reference image u. Image u is both like and different from the noisy image I0, i.e., the image intensity distributions are different, but the edge directions and texture information are similar.

**Block Diagram**

**Advantages**

Here, the auxiliary noise-free image has been used as a prior when we denoise one of the noisy images in the multicomponent remote-sensing image. The edge information of the reference image is fully considered, and a new smoothing term reference to the edges is concentrated.

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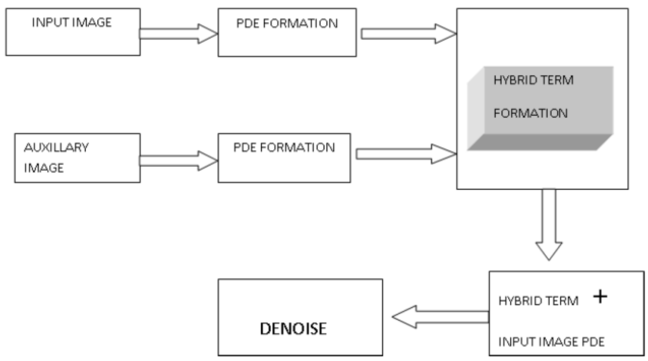



**Figure 2**

structed in the proposed method. Comprehensive experiments using different multispectral and hyperspectral images with different levels of noise were carried out. We have also compared the proposed method with other state-of-the-art methods, and the better performance of the proposed method is demonstrated. In particular, when the variance of the noise in the multi spectral In particular, when the variance of the noise in the multispectral image is large, the advantage of the proposed method is more obvious. Image is large, the advantage of the proposed method is more obvious.

## IV. Applications

The pde based denoising can be used mainly for hyperspectral images which has wide verities of applications in many fields. This technique can be used for any fields where multispectral or hyperspectral imaging is used. Hyperspectral remote sensing is used in a wide array of applications. Although originally developed for mining and geology (the ability of hyperspectral imaging to identify various minerals makes it ideal for the mining and oil industries, where it can be usedto look for ore and oil),[2][5] it has now spread into fields as widespread as ecology

**Food Processing.**

Hyperspectral image of" sugar end" potato strips shows invisible defects In the food processing industry, hyperspectral imaging, combined with intelligent software, enables digital sorters (also called optical sorters) to identify and remove defects and foreign material (FM) that are invisible to traditional camera and laser sorters.[10] By improving the accuracy of defect and FM removal, the food processors objective is to enhance product quality and increase yields.

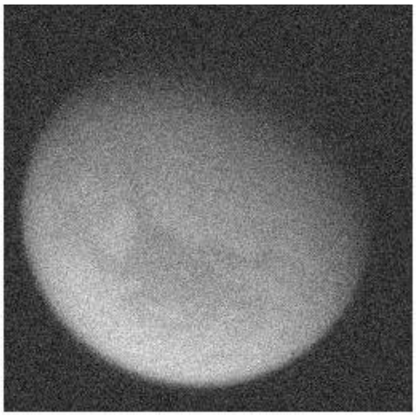
**Mineralogy**

Hyperspectral remote sensing of minerals is well developed. Many minerals can be identified from airborne images, and their relation to the presence of valuable minerals, such as gold and diamonds, is well understood. Currently, progress is towards understanding the relationship between oil and gas leakages from pipelines and natural wells, and their effects on the vegetation and the spectral signatures.

Environment Most countries require continuous monitoring of emissions produced by coal and oil-fired power plants, municipal and hazardous waste incinerators, cement plants, as well as many other types of industrial sources. This monitoring is usually performed using extractive sampling systems coupled with infrared spectroscopy techniques. Some recent standoff measurements performed allowed the evaluation of the air quality but not many remote independent methods allow for low uncertainty measurements

## V. Results

Denoised an image using simple partial differential equation. Then made an auxiliary
Image by adding noise manually and denoise the auxiliary image using partial differential equations.
After that add the two denoised images to get another denoised image.



Orginal image

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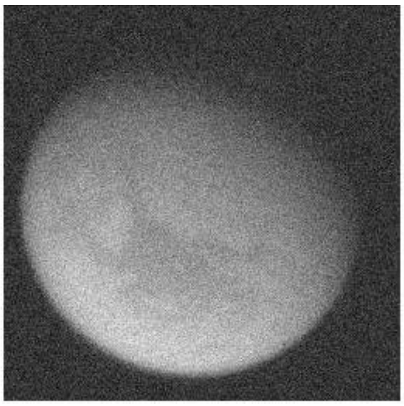
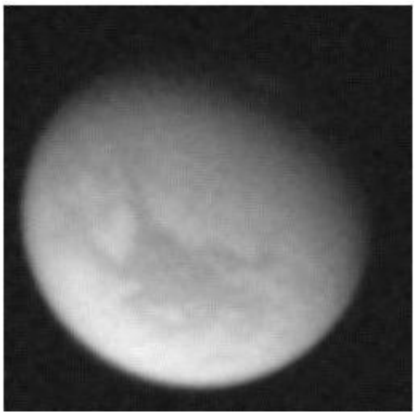
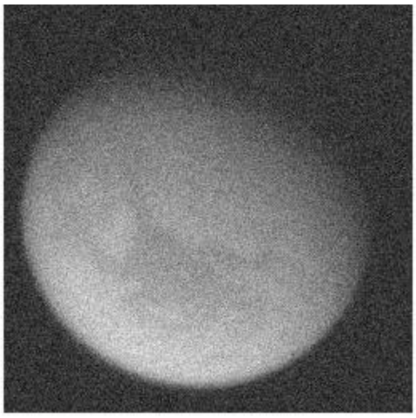
Auxiliary image



Resultant denoised image



Noisy image



Denoised image

## VI. Conclusion

All the denoising methods make use of the noisy image itself for denoising. So, smoothening may result in loss of information and blurring of edges. The auxiliary noise-free image has been used as a prior when we denoise one of the noisy images in the multicomponent remote sensing image. The edge information of the reference image is fully considered, and a new smoothing term reference to the edges is constructed in the proposed method. Comprehensive experiments using different multispectral and hyperspectral images with different levels of noise were carried out. We have also compared the proposed method with other state-of-the art methods, and the better performance of the proposed method is demonstrated. In particular, when the variance of the noise in the multispectral image is large, the advantage of the proposed method is more obvious.

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